Dear Reader,
Thank you for purchasing early access to *Classic Computer Science Problems in Python*. Python is a popular, elegant, and easy-to-learn language that attracts developers from a variety of backgrounds. I believe the problems in this intermediate book will help seasoned programmers learn the language, and new programmers accelerate their CS education. This book covers such a diversity of problem solving techniques, that there is truly something for everyone. However, basic knowledge of the Python language is assumed. This is a great second book on Python, but not a book for complete beginners.

Chapter 1, *Small Problems*, introduces problem solving techniques that will likely look familiar to most readers. Things like recursion, memoization, and simulation are essential building blocks of other techniques that are explored in later chapters. We follow this gentle introduction with Chapter 2, *Search Problems*. Search is such a large topic that you could arguably place most problems in the entire book under its banner. Our goal in chapter 2 is to introduce the most essential search algorithms including binary search, depth-first search, breadth-first search, and A*. These algorithms are reused throughout the rest of the book.

In Chapter 3, *Constraint Satisfaction Problems*, we build a framework for solving a broad range of problems that can be abstractly defined by variables of limited domains that have constraints between them. This includes such classic problems as The Eight Queens Problem, The Australian Map Coloring Problem, and the crypto-arithmetic SEND+MORE=MONEY.

Chapter 4, *Graph Problems*, explores the world of graph algorithms, which to the uninitiated are surprisingly broad in their applicability. In the chapter, we build a graph data structure and then use it to solve several classic problems. Chapter 5, *Genetic Algorithms*, explores a technique that is less deterministic than most covered in the book, but sometimes can solve a problem traditional algorithms cannot in a reasonable amount of time.

Chapter 6, *K-Means Clustering*, is perhaps the most algorithmically specific chapter in the book. This clustering technique is simple to implement, easy to understand, and broadly applicable. Chapter 7, *Fairly Simple Neural Networks*, aims to explain what a neural network is, and give the reader a taste of what a very simple neural network looks like. It does not aim to provide comprehensive coverage of this exciting and evolving field.

Chapter 8 looks at *Adversarial Search* techniques for creating artificial opponents for 2-player perfect information games like checkers, chess, and connect four. Finally, Chapter 9, *Miscellaneous Problems*, covers interesting (and fun) problems that didn’t quite fit anywhere else in the book.

Please note that since the book is still in development, you will be joining me on this exciting journey. I look forward to your feedback.

Thank you again,
—David Kopec (david@oaksnow.com)
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Introduction

Thank you for purchasing Classic Computer Science Problems in Python. Python is one of the most popular programming languages in the world. People become Python programmers from a variety of backgrounds. Some have a formal computer science education. Others learn Python as a hobby. Still others use Python in a professional setting but their primary job is not to be a software developer. The problems in this intermediate book will help seasoned programmers refresh on ideas from their CS education while learning some advanced features of the language. Self-taught programmers will accelerate their CS education by learning classic problems in the language of their choice—Python. This book covers such a diversity of problem-solving techniques that there is truly something for everyone.

This book is not an introduction to Python. There are numerous excellent books from Manning and other publishers in that vein. Instead, this book assumes that you are already an intermediate–advanced Python programmer. Although this book requires Python 3.7, mastery of every facet of the latest version of Python is not assumed. In fact, the book’s content was created with the assumption that it would serve as learning material to help one achieve such mastery. On the other hand, this book is not appropriate for readers completely new to Python.

0.1 Why Python?

Python is used in pursuits as diverse as data science, film-making, computer science education, IT management, and much more. There really is no computing field that Python has not touched (except maybe kernel development). Python is loved for its flexibility, beautiful and

1 If you are just starting your Python journey, you may want to first checkout The Quick Python Book, Third Edition, by Naomi Ceder (Manning, 2018) before beginning this book.
succinct syntax, object-oriented purity, and bustling community. The strong community is important because it means Python is welcoming to newcomers and has a large ecosystem of available libraries for developers to build upon.

For the above reasons Python is sometimes thought of as a “beginner-friendly” language. And that characterization is probably true. Most people would agree that Python is easier to learn than C++ for example, and its community is almost certainly friendlier to newcomers. So, many people learn Python because it is approachable, and they start writing the programs they want to write fairly quickly. However, they may never have received an education in computer science that teaches them all of the powerful problem-solving techniques available to them. If you are one of those programmers, who knows Python, but does not know CS, then this book is for you.

Other people learn Python as a second, third, fourth, or fifth language after a long time working in software development. For them, seeing old problems they’ve already seen in another language will help them accelerate their learning of Python. For them, this book may be a good refresher before a job interview, or it might expose them to some problem-solving techniques they had not previously thought of exploiting in their work. I would encourage them to skim the table-of-contents to see if there are topics in this book that excite them.

0.2 What is a classic computer science problem?

Some say that computers are to computer science as telescopes are to astronomy. If that’s the case, then is a programming language like a telescope lens? In any event, the term “computer science problems” is used here to mean “programming problems typically taught in an undergraduate computer science curriculum.”

There are certain programming problems that are given to new programmers to solve, whether in a classroom setting during the pursuit of a bachelor’s degree (in computer science, software engineering, etc.) or within the confines of an intermediate programming textbook (for example, a first book on artificial intelligence or algorithms), that have become commonplace enough to be deemed “classic.” A selection of such problems is what you will find in this book.

The problems range from the trivial, which can be solved in a few lines of code, to the complex, which require the buildup of systems over multiple chapters. Some problems touch on artificial intelligence, and others simply require common sense. Some problems are practical, and other problems are fanciful.

0.3 What kinds of problems are in this book?

Chapter 1 introduces problem-solving techniques that will likely look familiar to most readers. Things like recursion, memoization, and bit manipulation are essential building blocks of other techniques explored in later chapters.

This gentle introduction is followed by chapter 2, which focuses on search problems. Search is such a large topic that you could arguably place most problems in the book under its banner.
Chapter 2 introduces the most essential search algorithms, including binary search, depth-first search, breadth-first search, and A*. These algorithms are reused throughout the rest of the book.

In chapter 3, you will build a framework for solving a broad range of problems that can be abstractly defined by variables of limited domains that have constraints between them. This includes such classics as the eight queens problem, the Australian map-coloring problem, and the cryptarithmetic SEND+MORE=MONEY.

Chapter 4 explores the world of graph algorithms, which to the uninitiated are surprisingly broad in their applicability. In this chapter, you will build a graph data structure and then use it to solve several classic optimization problems.

Chapter 5 explores genetic algorithms, a technique that is less deterministic than most covered in the book, but that sometimes can solve some problem traditional algorithms cannot solve in a reasonable amount of time.

Chapter 6 covers k-means clustering and is perhaps the most algorithmically specific chapter in the book. This clustering technique is simple to implement, easy to understand, and broadly applicable.

Chapter 7 aims to explain what a neural network is, and to give the reader a taste of what a very simple neural network looks like. It does not aim to provide comprehensive coverage of this exciting and evolving field. In this chapter, you will build a neural network from first principles, using no external libraries, so you can really see how a neural network works.

Chapter 8 is on adversarial search in two-player perfect information games. You will learn a search algorithm known as minimax which can be used to develop an artificial opponent that can play games like chess, checkers, and Connect Four well.

Finally, chapter 9 covers interesting (and fun) problems that did not quite fit anywhere else in the book.

0.4 Who is this book for?

This book is for both intermediate and experienced programmers. Experienced programmers who want to deepen their knowledge of Python will find comfortably familiar problems from their computer science or programming education. Intermediate programmers will be introduced to these classic problems in the language of their choice—Python. Developers getting ready for coding interviews will likely find this book to be valuable preparation material.

In addition to professional programmers, students enrolled in undergraduate computer science programs who have an interest in Python will likely find this book helpful. It makes no attempt to be a rigorous introduction to data structures and algorithms. This is not a data structures and algorithms textbook—you will not find proofs or extensive use of big-O notation within its pages. Instead, it is positioned as an approachable, hands-on tutorial to the problem-solving techniques that should be the end product of taking data structure, algorithm, and artificial intelligence classes.
Once again, knowledge of Python’s syntax and semantics is assumed. A reader with zero programming experience will get little out of this book. And a programmer with zero Python experience will almost certainly struggle. In other words, *Classic Computer Science Problems in Python* is a book for working Python programmers and computer science students.

### 0.5 Python versioning, source code repository, and type hints

The source code in this book was written to adhere to version 3.7 of the Python language. We did use features of Python that only became available in Python 3.7, so some of the code will not run on earlier versions of Python. Instead of struggling, trying to make the examples run in an earlier version, please just download the latest version of Python before starting the book.

We only make use of the Python standard library (with a slight exception in Chapter 2, where we install the `typing_extensions` module), so all of the code in this book should run on any platform where Python is supported (macOS, Windows, GNU/Linux, etc.). The code in this book was only tested against CPython (the main Python interpreter available from python.org), although it is likely most of it will run in a Python 3.7 compatible version of another Python interpreter.

This book does not explain how to use Python tools like editors, IDEs, debuggers, and the Python REPL. All of the source code from the book is available online from the GitHub repository [https://github.com/davecom/ClassicComputerScienceProblemsInPython](https://github.com/davecom/ClassicComputerScienceProblemsInPython). The source code is organized into folders by chapter. As you read each chapter you will see the name of a source file in the header of each code listing. You can find that source file in its respective folder in the repository. You should be able to run the problem by just entering “python3 filename.py” or “python filename.py” depending on your computer's setup with regards to the name of the Python 3 interpreter.

Every code listing in this book makes use of Python type hints, also known as type annotations. These annotations are a relatively new feature for the Python language, and they may look intimidating to Python programmers who have never seen them before. They are used for three reasons:

1. They provide clarity about the types of variables, function parameters, and function returns.
2. They self-document the code in a sense as a result of reason 1. Instead of having to search through a comment or docstring to find the return type of a function, you can just look at its signature.
3. They allow the code to be type-checked for correctness. One popular Python type checker is `mypy`.

Not everyone is a fan of type hints and choosing to use them throughout the book was frankly a gamble. Hopefully they will be a help instead of a hindrance. It takes a little more time to write Python with type hints, but it provides more clarity when read back. An interesting note is that type hints have no effect on the actual running of the code in the Python interpreter. You
can remove the type hints from any of the code in this book and it should still run. If you have never seen type hints before and feel you need a more comprehensive introduction to them before diving into the book please see Appendix C, which provides a crash-course in type hints.

0.6 No graphics, no UI code, just the standard library

There are no examples in this book that produce graphical output or that make use of a graphical user interface (GUI). Why? The goal is to solve the posed problems with solutions that are as concise and readable as possible. Often, doing graphics gets in the way, or makes solutions significantly more complex than they need to be to illustrate the technique or algorithm in question.

Further, by not making use of any GUI framework, all of the code in the book is eminently portable. It can as easily run on an embedded distribution of Python running on Linux, as it can on a desktop running Windows. Also, a conscious decision was made to only use packages from the Python standard library instead of any external libraries as most advanced Python books do. Why? The goal is to teach problem solving techniques from first principles, not to “pip install a solution.” By having to work through every problem from scratch, you will hopefully gain an understanding about how popular libraries work behind the scenes. At a minimum, only using the standard library makes the code in this book more portable and easier to run for the reader.

This is not to say that graphical solutions are not sometimes more illustrative of an algorithm than text-based solutions. It simply was not the focus of this book. It was another layer of complexity we did not choose to address.

0.7 Part of a series

This is the second book in a series of books I am writing titled “Classic Computer Science Problems” and published by Manning. The first book was Classic Computer Science Problems in Swift published in 2018. In each book in the series we aim to provide language specific insight, while learning through the lens of the (mostly) same computer science problems.

If you enjoy this book and plan to learn another language covered by the series, you may find going from one book to another an easy way to improve your mastery of that language. For now, the series covers just Swift and Python. I wrote the first two books myself since I have significant experience in both of those languages, but we are already discussing plans for future books in the series co-authored by people who are experts in other languages. I encourage you to look out for them if you enjoy this book. For more information about the series, visit https://classicproblems.com.
To get started, we will explore some simple problems that can be solved with no more than a few relatively short functions. Although these problems are small, they will still allow us to explore some interesting problem-solving techniques. Think of them as a good warmup.

1.1 The Fibonacci sequence

The Fibonacci sequence is a sequence of numbers such that any number, except for the first and second, is the sum of the previous two:

\[0, 1, 1, 2, 3, 5, 8, 13, 21\ldots\]

The value of the first Fibonacci number in the series is 0. The value of the fourth Fibonacci number is 2. It follows that to get the value of any Fibonacci number, \(n\), in the series, one can use the formula

\[\text{fib}(n) = \text{fib}(n - 1) + \text{fib}(n - 2)\]

1.1.1 A first recursive attempt

The preceding formula for computing a number in the Fibonacci sequence (illustrated in figure 1.1), a form of pseudocode, can be trivially translated into a recursive Python function (a recursive function is a function that calls itself). This mechanical translation will serve as the first version of our attempt at writing a function to return a given value of the Fibonacci sequence:
Listing 1.1 fib1.py

def fib1(n: int) -> int:
    return fib1(n - 1) + fib1(n - 2)

Figure 1.1 The height of each stickman is the addition of the previous two stickmen’s heights added together.

Let’s try to run this function by calling it with a value:

Listing 1.2 fib1.py continued

if __name__ == "__main__":
    print(fib1(5))

Uh, oh! If we try to run fib1.py, we generate an error:

RecursionError: maximum recursion depth exceeded

The issue is that fib1() will run forever without returning a final result. Every call to fib1() results in another two calls of fib1() with no end in sight. We call such a circumstance infinite recursion, and it is analogous to an infinite loop.
1.1.2 Utilizing base cases

Notice that until we run `fib1()`, there is no indication from your Python environment that there is anything wrong with it. It is the duty of the programmer to avoid infinite recursion, not the compiler or the interpreter. The reason for the infinite recursion is that we never specified a base case. In a recursive function, a base case serves as a stopping point.

In the case of the Fibonacci function, we have natural base cases in the form of the special first two sequence values, 0 and 1. Neither 0 nor 1 is the sum of the previous two numbers in the sequence. Instead, they are the special first two values. Let’s try specifying them as base cases:

```
Listing 1.3 fib2.py

def fib2(n: int) -> int:
    if n < 2:  # base case
        return n
    return fib2(n - 2) + fib2(n - 1)  # recursive case
```

In recursion, we go around and around...
NOTE The fib2() version of the Fibonacci function returns 0 as the zeroth number (fib2(0)), rather than the first number, as in our original proposition. In a programming context, this kind of makes sense because we are used to sequences starting with a zeroth element.

fib2() can be called successfully and will return correct results. Try calling it with some small values:

Listing 1.4 fib2.py continued

```python
if __name__ == '__main__':
    print(fib2(5))
    print(fib2(10))
```

Do not try calling fib2(50). It will never finish executing! Why? Every call to fib2() results in two more calls to fib2() by way of the recursive calls fib2(n - 1) and fib2(n - 2) (see figure 1.3). In other words, the call tree grows exponentially. For example, a call of fib2(4) results in this entire set of calls:

```
fib2(4) -> fib2(3), fib2(2)
fib2(3) -> fib2(2), fib2(1)
fib2(2) -> fib2(1), fib2(0)
fib2(2) -> fib2(1), fib2(0)
fib2(1) -> 1
fib2(1) -> 1
fib2(1) -> 1
fib2(0) -> 0
fib2(0) -> 0
```

Figure 1.3 Every non-base-case call of fib2() results in two more calls of fib2().
If you count them (and as you can see if you add some print calls), there are 9 calls to \texttt{fib2()} just to compute the 4th element! It gets worse. There are 15 calls required to compute element 5, 177 calls to compute element 10, and 21,891 calls to compute element 20. We can do better.

### 1.1.3 Memoization to the rescue

Memoization is a technique in which you store the results of computational tasks when they are completed, so that when you need them again, you can look them up instead of needing to compute them a second (or millionth) time (see figure 1.4).²

![Figure 1.4 The human memoization machine](image)

Let's create a new version of the Fibonacci function that utilizes a Python dictionary for memoization purposes.

```python
from typing import Dict

memo: Dict[int, int] = {0: 0, 1: 1} # our base cases
```

²Donald Michie, a famous British computer scientist, coined the term memoization. Donald Michie, \textit{Memo functions: a language feature with “rote-learning” properties} (Edinburgh University, Department of Machine Intelligence and Perception, 1967).

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```python
def fib3(n: int) -> int:
    if n not in memo:
        memo[n] = fib3(n - 1) + fib3(n - 2)  # memoization
    return memo[n]

You can now safely call `fib3(50)`.

### Listing 1.6 fib3.py continued
```python
if __name__ == "__main__":
    print(fib3(5))
    print(fib3(50))
```

A call to `fib3(20)` will result in just 39 calls of `fib3()` as opposed to the 21,891 of `fib2()` resulting from the call `fib2(20)`. `memo` is prefilled with the earlier base cases of 0 and 1, saving `fib3()` from the complexity of another if statement.

#### 1.1.4 Automatic memoization

`fib3()` can be further simplified. Python has a built-in decorator for memoizing any function automagically. In `fib4()`, the decorator `@functools.lru_cache()` is used with the same exact code as we used in `fib2()`. Each time `fib4()` is executed with a novel argument, the decorator causes the return value to be cached. Upon future repeat calls of `fib4()` with the same argument, the previous return value of `fib4()` for that argument is retrieved from the cache and returned.

### Listing 1.7 fib4.py
```python
from functools import lru_cache

@lru_cache(maxsize=None)
def fib4(n: int) -> int:  # same definition as fib2()
    if n < 2:  # base case
        return n
    return fib4(n - 2) + fib4(n - 1)  # recursive case

if __name__ == "__main__":
    print(fib4(5))
    print(fib4(50))
```

Note that we are able to calculate `fib4(50)` instantly, even though the body of the Fibonacci function is the same as that in `fib2()`. `@lru_cache`’s maxsize property indicates how many of the most recent calls of the function it is decorating should be cached. Setting it to `None` indicates that there is no limit.
1.1.5 Keep it simple, Fibonacci

There is an even more performant option. We can solve Fibonacci with an old fashioned iterative approach.

Listing 1.8 fib5.py

```python
def fib5(n: int) -> int:
    if n == 0:
        return n  # special case
    last: int = 0  # initially set to fib(0)
    next: int = 1  # initially set to fib(1)
    for _ in range(1, n):
        last, next = next, last + next
    return next

if __name__ == '__main__':
    print(fib5(5))
    print(fib5(50))
```

**WARNING** The body of the `for` loop in `fib5()` uses tuple unpacking in perhaps a bit of an overly clever way. Some may feel that it sacrifices readability for conciseness. Others may find the conciseness in and of itself more readable. The gist is, `last` is being set to the previous value of `next`, and `next` is being set to the previous value of `last` plus the previous value of `next`. This avoids the creation of a temporary variable to hold the old value of `next` after `last` is updated, but before `next` is updated. Using tuple unpacking in this fashion for some kind of variable swap is common in Python.

With this approach, the body of the `for` loop will only run a maximum of `n - 1` times. In other words, this is the most efficient version yet. Compare 19 runs of the `for` loop body to 21,891 recursive calls of `fib2()` for the 20th Fibonacci number. That could make a serious difference in a real-world application!

In the recursive solutions, we worked backward. In this iterative solution, we work forward. Sometimes recursion is the most intuitive way to solve a problem. For example, the meat of `fib1()` and `fib2()` is pretty much a mechanical translation of the original Fibonacci formula. However, naive recursive solutions can also come with significant performance costs. Remember, any problem that can be solved recursively can also be solved iteratively.

1.1.6 Generating Fibonacci numbers with a generator

So far, we have written functions that output a single value in the Fibonacci sequence. What if we want to output the entire sequence up to some value instead? It is easy to convert `fib5()` into a Python generator using the `yield` statement. When the generator is iterated, each iteration will spew a value from the Fibonacci sequence using a `yield` statement.

Listing 1.9 fib6.py

```python
from typing import Generator
```
def fib6(n: int) -> Generator[int, None, None]:
    yield 0  # special case
    if n > 0: yield 1  # special case
    last: int = 0  # initially set to fib(0)
    next: int = 1  # initially set to fib(1)
    for _ in range(1, n):
        last, next = next, last + next
        yield next  # main generation step

if __name__ == "__main__":
    for i in fib6(50):
        print(i)

If you run fib6.py, you will see 51 numbers in the Fibonacci sequence printed. For each
iteration of the for-loop for i in fib6(50):, fib6() runs through to a yield statement. If
the end of the function is reached and there are no more yield statements, then the loop
finishes iterating.

## 1.2 Trivial compression

Saving space (virtual or real) is often important. It is more efficient to use less space, and it
can save money. If you are renting an apartment that is bigger than you need for your things
and family, then you may “downsize” to a smaller place that is less expensive. If you are
paying by the byte to store your data on a server, then you may want to compress it so that its
storage costs you less. Compression is the act of taking data and encoding it (changing its
form) in such a way that it takes up less space. Decompression is reversing the process,
returning the data to its original form.

If it is more storage-efficient to compress data, then why is all data not compressed? There
is a tradeoff between time and space. It takes time to compress a piece of data and to
decompress it back into its original form. Therefore, data compression only makes sense in
situations where small size is prioritized over fast execution. Think of large files being
transmitted over the internet. Compressing them makes sense because it will take longer to
transfer the files than it will to decompress them once received. Further, the time taken to
compress the files for their storage on the original server only needs to be accounted for once.

The easiest way to compress data is to realize that its storage type uses more bits than are
strictly required for its contents. For instance, thinking low-level, if an unsigned integer that
will never exceed 65,535 is being stored as a 64-bit unsigned integer in memory, it is being
stored inefficiently. It could instead be stored as a 16-bit unsigned integer. This would reduce
the space consumption for the actual number by 75% (16 bits instead of 64 bits). If there are
millions of such numbers being stored inefficiently, it can add up to megabytes of wasted
space.
In Python, sometimes for the sake of simplicity, which is a legitimate goal of course, the developer is shielded from thinking in bits. There is no 64-bit unsigned integer type, and there is no 16-bit unsigned integer type. There is just a single `int` type that can store numbers of arbitrary precision. The function `sys.getsizeof()` can help you find how many bytes of memory your Python objects are consuming. However, due to the inherent overhead of the Python object system, there is no way to create an `int` that takes up less than 28 bytes (224 bits) in Python 3.7. A single `int` can be extended one bit at a time (as we will do in this example), but it consumes a minimum of 28 bytes.

**NOTE** If you are a little rusty regarding binary, recall that a bit is a single value that is either a 1 or a 0. A sequence of 1s and 0s is read in base 2 to represent a number. For the purposes of this section, you do not need to do any math in base 2, but you do need to understand that the number of bits that a type stores determines how many different values it can represent. For example, 1 bit can represent 2 values (0 or 1), 2 bits can represent 4 values (00, 01, 10, 11), 3 bits can represent 8 values, and so on.

If the number of possible different values that a type is meant to represent is less than the number of values that the bits being used to store it can represent, it can likely be more efficiently stored. Consider the nucleotides that form a gene in DNA. Each nucleotide can only be one of four values: A, C, G, or T (there will be more about this in chapter 2). Yet, if the gene is stored as a `str`, which can be thought of as a collection of Unicode characters, each nucleotide will be represented by a character, which generally requires 8 bits of storage. In binary, just 2 bits are needed to store a type with four possible values: 00, 01, 10, and 11 are the four different values that can be represented by 2 bits. If A is assigned 00, C is assigned 01, G is assigned 10, and T is assigned 11, then the storage required for a string of nucleotides can be reduced by 75% (8 bits to 2 bits per nucleotide).

Instead of storing our nucleotides as a `str`, they can be stored as a *bit string* (see figure 1.5). A bit string is exactly what it sounds like—an arbitrary length sequence of 1s and 0s. Unfortunately, the Python standard library contains no off-the-shelf construct for working with bit strings of arbitrary length. The following code converts a `str` composed of As, Cs, Gs, and Ts into a string of bits and back again. The string of bits is stored within an `int`. Since the `int` type in Python can be of any length, it can be used as a bit string of any length. To convert back into a `str`, we will implement the Python `__str__()` special method.

---

3This example is inspired by *Algorithms, 4th Edition* by Robert Sedgewick and Kevin Wayne (Addison-Wesley Professional, 2011), page 819.

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Figure 1.5 Compressing a String representing a gene into a 2-bit-per-nucleotide bit string.

Listing 1.10 trivial_compression.py

class CompressedGene:
    def __init__(self, gene: str) -> None:
        self._compress(gene)

A CompressedGene is provided a str of characters representing the nucleotides in a gene and internally stores the sequence of nucleotides as a bit string. The __init__() method’s main responsibility is to initialize the bit-string construct with the appropriate data. __init__() calls _compress() to do the dirty work of actually converting the provided str of nucleotides into a bit string. Note that _compress() starts with an underscore. Python has no concept of truly private methods/variables (all variables/methods can be accessed through reflection, there’s no strict enforcement of privacy). A leading underscore is used as a convention to indicate the implementation of a method should not be relied on by actors outside of the class (it is subject to change and should be treated as private).

Next, let’s look at how we can actually perform the compression.

TIP If you start a method or instance variable name in a class with two leading underscores, Python will “name mangle” it, changing its implementation name with a salt and not making it easily discoverable by other classes. We use one underscore in this book to indicate a “private” variable or method, but you may wish to use two if you really want to emphasize that something is private. For more on naming in Python, checkout the
The \_compress() method looks at each character in the \texttt{str} of nucleotides sequentially. When it sees an A, it adds 00 to the bit string. When it sees a C, it adds 01. And so on. Remember that 2 bits are needed for each nucleotide. As a result, before we add each new nucleotide, we shift the bit string two bits to the left (\texttt{self.bit_string} <<= 2). Every nucleotide is added using an “or” operation (\texttt{|}). After the left shift, two 0s are added to the right-hand side of the bit string. In bitwise operations, “oring” (ex. \texttt{self.bit_string} \texttt{|=} \texttt{0b10}) 0s with any other value results in the other value replacing the 0s. In other words, we continually add two new bits to the right-hand side of the bit string. The two bits that are added are determined by the type of nucleotide.

Finally, we will implement decompression and the special \_\texttt{str}() method that uses it.
decompress() reads 2 bits from the bit string at a time. It uses those two bits to determine which character to add to the end of the str representation of the gene. Since the bits are being read in the opposite order from that which they were compressed in (right to left instead of left to right), the str representation is ultimately reversed (using the slicing notation for reversal [:::-1]). Finally, note how the convenient int method bit_length() aided in the development of decompress(). Let’s test it out.

Listing 1.13 trivial_compression.py continued

```python
if __name__ == '__main__':
    from sys import getsizeof
    original: str =
    "TAGGGATTAACCGTTATATATATAGCCATGGATCGATTATATAGGGATTAACCGTTATATATATATAGCCATGGATCGATTATA" * 100
    print("original is {} bytes".format(getsizeof(original)))
    compressed: CompressedGene = CompressedGene(original)  # compress
    print("compressed is {} bytes".format(getsizeof(compressed.bit_string)))
    print(compressed)  # decompress
    print("original and decompressed are the same: {}".format(original == compressed.decompress()))
```

Using the `sys.getsizeof()` method, we can indicate in the output if we did indeed save almost 75% of the memory cost of storing the gene through our compression scheme.

Listing 1.14 trivial_compression.py output

```
original is 8649 bytes
compressed is 2320 bytes
TAGGGATTAACCGTTATATATATAGCCATGGATCGATTATATAGGGATTAACCGTTATATATATATAGCCATGGATCGATTATA
original and decompressed are the same: True
```

**NOTE** In the CompressedGene class, we used if-statements extensively to decide between a series of cases in both the compression and the decompression methods. Since Python has no switch-statement, this is somewhat typical. What you will also see in Python sometimes is a high reliance on dictionaries in place of extensive if-statements to deal with a set of cases. Imagine for instance if we had a dictionary from which we could lookup each nucleotide’s respective bits. This can sometimes be more readable, but it can come with a performance cost. Even though a dictionary lookup is technically O(1), in reality due to the cost of running a hash function, it will sometimes be less performant to use a dictionary in place of a series of ifs. Of course, whether this holds will depend on what a particular program’s if-statements actually need to evaluate to make their decision. You may want to run performance tests on both methods if you need to make a decision between ifs and dictionary lookup in a particularly critical section of code.
1.3 Unbreakable encryption

A one-time pad is a way of encrypting a piece of data by combining it with meaningless random dummy data in such a way that the original cannot be reconstituted without access to both the product and the dummy data. In essence, this leaves the encrypter with a key pair (one key is the product, one key is the random dummy data). One key on its own is useless—only the combination of both keys can unlock the original data. When performed correctly, a one-time pad is a form of unbreakable encryption. Figure 1.6 shows the process.

![Diagram of one-time pad encryption process]

**Figure 1.6** A one-time pad results in two keys that can be separated and then recombined to recreate the original data.

1.3.1 Getting the data in order

In this example, we will encrypt a `str` using a one-time pad. One way of thinking about a Python 3 `str` is as a sequence of UTF-8 bytes (with UTF-8 being a Unicode character encoding). A `str` can be converted into a sequence of UTF-8 bytes (represented as the `bytes` type) through the `encode()` method. Likewise, a sequence of UTF-8 bytes can be converted back into a `str` using the `decode()` method on the `bytes` type.

There are three criteria that the dummy data used in a one-time pad encryption operation must meet for the resulting product to be unbreakable. The dummy data must be the same length as the original data, truly random, and completely secret. The first and third criteria make common sense. If the dummy data repeats, because it is too short, there could be an observed pattern. If one of the keys is not truly secret (perhaps it is reused elsewhere or...
partially revealed), then an attacker has a clue. The second criteria poses a question all its own—can we produce truly random data? The answer for most computers is no.

In this example we will use the pseudo-random data generating function `token_bytes()` from the `secrets` module (first included in the standard library in Python 3.6). Our data will not be truly random (but close enough for our purposes), in the sense that the `secrets` package still is using a pseudo-random number generator behind the scenes. Let’s work on generating a random key for use as dummy data.

```python
from secrets import token_bytes
from typing import Tuple

def random_key(length: int) -> int:
    # generate length random bytes
    tb: bytes = token_bytes(length)
    # convert those bytes into a bit string and return it
    return int.from_bytes(tb, "big")
```

This function creates an `int` filled with `length` random bytes. The method `int.from_bytes()` is used to convert from `bytes` to `int`. How can multiple bytes be converted to a single integer? The answer lies in our last problem, “Trivial compression.” In that example, we learned that the `int` type can be of arbitrary size and we saw how it can be used as a generic bit string. `int` is being used in the same way here. For example, the `from_bytes()` method will take 7 bytes (7 bytes * 8 bits = 56 bits) and convert it into a 56 bit integer. Why is this useful? Bitwise operations can be executed more easily and performantly on a single `int` (read long bit string) than on many individual bytes in a sequence. And we are about to use the bitwise operation, XOR.

### 1.3.2 Encrypting and decrypting

How will the dummy data be combined with the original data that we want to encrypt? The XOR operation will serve this purpose. XOR is a logical bitwise (operates at the bit level) operation that returns true when one of its operands is true, but returns false when both are true or neither is true. As you may have guessed, XOR stands for exclusive or.

In Python, the XOR operator is `^`. In the context of the bits of binary numbers, XOR returns 1 for `0 ^ 1` and `1 ^ 0`, but 0 for `0 ^ 0` and `1 ^ 1`. If the bits of two numbers are combined using XOR, a helpful property is that the product can be recombined with either of the operands to produce the other operand.

<table>
<thead>
<tr>
<th>A</th>
<th>^</th>
<th>B</th>
<th>=</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>^</td>
<td>B</td>
<td>=</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>^</td>
<td>A</td>
<td>=</td>
<td>B</td>
</tr>
</tbody>
</table>
This key insight forms the basis of one-time pad encryption. To form our product, we will simply XOR an `int` representing the bytes in our original `str` with a randomly generated `int` of the same bit length (as produced by `random_key()`). Our returned key pair will be the dummy data and the product.

### Listing 1.16 unbreakable_encryption.py continued

```python
def encrypt(original: str) -> Tuple[int, int]:
    original_bytes: bytes = original.encode()
    dummy: int = random_key(len(original_bytes))
    original_key: int = int.from_bytes(original_bytes, "big")
    encrypted: int = original_key ^ dummy  # XOR
    return dummy, encrypted
```

**NOTE** `int.from_bytes()` is being passed two arguments. The first is the `bytes` that we want to convert into an `int`. The second is the endianness of those `bytes` ("big"). Endianness refers to the byte-ordering used to store data. Does the most significant byte come first or does the least significant byte come first? In our case, it does not matter, as long as we use the same ordering both when we encrypt and we decrypt since we are actually only manipulating the data at the individual bit level. However, in other situations, when you are not controlling both ends of the encoding process, the ordering can absolutely matter, so be careful!

Decryption is simply a matter of recombining the key pair we generated with `encrypt()`. This is achieved once again by doing an XOR operation between each and every bit in the two keys. The ultimate output must be converted back to a `str`. First, the `int` is converted to `bytes` using `int.to_bytes()`. This method requires the number of bytes to be converted from the `int`. To get this number, we divide the bit length by eight (the number of bits in a byte). Finally, the `bytes` method `decode()` gives us back a `str`.

### Listing 1.17 unbreakable_encryption.py continued

```python
def decrypt(key1: int, key2: int) -> str:
    decrypted: int = key1 ^ key2  # XOR
    temp: bytes = decrypted.to_bytes((decrypted.bit_length() + 7) // 8, "big")
    return temp.decode()
```

It was necessary to add 7 to the length of the decrypted data before using integer-division (`//`) to divide by 8 to ensure that we “round up,” to avoid an off-by-one error. If our one-time pad encryption truly works, we should be able to encrypt and decrypt the same Unicode string without issue.

### Listing 1.18 unbreakable_encryption.py continued

```python
if __name__ == "__main__":
    key1, key2 = encrypt("One Time Pad!")
    result: str = decrypt(key1, key2)
    print(result)
```
If your console outputs One Time Pad! then everything worked.

1.4 Calculating pi

The mathematically significant number pi (π or 3.14159...) can be derived using many formulas. One of the simplest is the Leibniz formula. It posits that the convergence of the following infinite series is equal to pi:

\[ \pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11}... \]

You will notice that the infinite series’ numerator remains 4 while the denominator increases by 2, and the operation on the terms alternates between addition and subtraction.

We can model the series in a straightforward way by translating pieces of the formula into variables in a function. The numerator can be a constant 4. The denominator can be a variable that begins at 1 and is incremented by 2. The operation can be represented as either -1 or 1 based on whether we are adding or subtracting. Finally, the variable `pi` is used in Listing 1.19 to collect the sum of the series as the for-loop proceeds.

```
Listing 1.19 calculating_pi.py

def calculate_pi(n_terms: int) -> float:
    numerator: float = 4.0
    denominator: float = 1.0
    operation: float = 1.0
    pi: float = 0.0
    for _ in range(n_terms):
        pi += operation * (numerator / denominator)
        denominator += 2.0
        operation *= -1.0
    return pi

if __name__ == "__main__":
    print(calculate_pi(1000000))
```

**TIP** On most platforms, Python floats are 64-bit floating point numbers (or double in C).

This function is an example of how rote conversion between formula and programmatic code can be both simple and effective in modeling or simulating an interesting concept. Rote conversion is a useful tool, but we must keep in mind that it is not necessarily the most efficient solution. Certainly, the Leibniz formula for pi can be implemented with more efficient or compact code.

**NOTE** The more terms in the infinite series (the higher the value of `n_terms` when `calculate_pi()` is called), the more accurate the ultimate calculation of pi will be.
1.5 The Towers of Hanoi

Three vertical pegs (henceforth “towers”) stand tall. We will label them A, B, and C. Donut-shaped discs are around tower A. The widest disc is at the bottom, and we will call it disc 1. The rest of the discs above disc 1 are labeled with increasing numerals and get progressively narrower. For instance, if we were to work with three discs, the widest disc, the one on the bottom, would be 1. The next widest disc, disc 2, would sit on top of disc 1. And finally, the narrowest disc, disc 3, would sit on top of disc 2. Our goal is to move all of the discs from tower A to tower C given the following constraints:

- Only one disc can be moved at a time.
- The topmost disc of any tower is the only one available for moving.
- A wider disc can never be atop a narrower disc.

Figure 1.7 summarizes the problem.

![The Towers of Hanoi diagram](image)

Figure 1.7 The challenge is to move the three discs, one at a time, from tower A to tower C. A larger disc may never be on top of a smaller disc.

1.5.1 Modeling the towers

A stack is a data structure that is modeled on the concept of Last-In-First-Out (LIFO). The last thing put into it is the first thing that comes out of it. The two most basic operations on a stack
are push and pop. A \textit{push} puts a new item into a stack, whereas a \textit{pop} removes and returns the last item put in. We can easily model a stack in Python using a \texttt{list} as a backing store.

\textbf{Listing 1.20 hanoi.py}

```python
from typing import TypeVar, Generic, List
T = TypeVar('T')

class Stack(Generic[T]):
    def \_init\_(self) -> None:
        self._container: List[T] = []
    def push(self, item: T) -> None:
        self._container.append(item)
    def pop(self) -> T:
        return self._container.pop()
    def \_repr\_(self) -> str:
        return repr(self._container)
```

\textbf{NOTE} This \texttt{Stack} class implements \texttt{\_repr\_(\_)} so that we can easily explore the contents of a tower. \texttt{\_repr\_(\_)} is what will be output when \texttt{print()} is applied to a \texttt{Stack}.

\textbf{NOTE} As was described in the introduction, this book utilizes type hints throughout. The import of \texttt{Generic} from the \texttt{typing} module, enables \texttt{Stack} to be generic over a particular type in type hints. The arbitrary type \texttt{T} is defined in \texttt{T = TypeVar('T')}. \texttt{T} can be any type. When a type hint is later used for a \texttt{Stack} to solve the Hanoi problem, it is type hinted as \texttt{type Stack[\texttt{int}]}, which means \texttt{T} is filled-in with \texttt{type int}. In other words, the stack is a stack of integers.

Stacks are perfect stand-ins for the towers in \textit{The Towers of Hanoi}. When we want to put a disc onto a tower, we can just push it. When we want to move a disc from one tower to another, we can pop it from the first and push it onto the second.

Let’s define our towers as \texttt{Stacks} and fill the first tower with discs.

\textbf{Listing 1.21 hanoi.py continued}

```python
num_discs: int = 3
tower_a: Stack[int] = Stack()
tower_b: Stack[int] = Stack()
tower_c: Stack[int] = Stack()
for i in range(1, num_discs + 1):
    tower_a.push(i)
```
1.5.2 Solving The Towers of Hanoi

How can The Towers of Hanoi be solved? Imagine we were only trying to move 1 disc. We would know how to do that, right? In fact, moving one disc is our base case for a recursive solution to The Towers of Hanoi. The recursive case is moving more than 1 disc. Therefore, the key insight is that we essentially have two scenarios we need to codify: moving 1 disc (the base case) and moving more than one disc (the recursive case).

Let’s look at a specific example to understand the recursive case. Say we have three discs (top, middle, and bottom) on tower A that we want to move to tower C (it may help to sketch out the problem as you follow along). We could first move the top disc to tower C. Then we could move the middle disc to tower B. Then we could move the top disc from tower C to tower B. Now we have the bottom disc still on tower A and the upper two discs on tower B. Essentially, we have now successfully moved two discs from one tower (A) to another tower (B). Moving the bottom disc from A to C is our base case (moving a single disc). Now we can move the two upper discs from B to C in the same procedure that we did from A to B. We move the top disc to A, the middle disc to C, and finally the top disc from A to C.

**TIP** In a computer science classroom, it is not uncommon to see a little model of the towers built using dowels and plastic donuts. You can build your own model using three pencils and three pieces of paper. It may help you visualize the solution.

In our three-disc example, we had a simple base case of moving a single disc, and recursive case of moving all of the other discs (two in this case), using the third tower temporarily. We could break the recursive case into three steps:

1. Move the upper n-1 discs from tower A to B (the temporary tower) using C as the in-between.
2. Move the single lowest disc from A to C.
3. Move the n-1 discs from tower B to C using A is the in-between.

The amazing thing is that this recursive algorithm not only works for three discs, but for any number of discs. We will codify it as a function called `hanoi()` that is responsible for moving discs from one tower to another, given a third temporary tower.

```
Listing 1.22 hanoi.py continued

def hanoi(begin: Stack[int], end: Stack[int], temp: Stack[int], n: int) -> None:
    if n == 1:
        end.push(begin.pop())
    else:
        hanoi(begin, temp, end, n - 1)
```

""About the Towers of Hanoi," in Surveying the Field of Computing by Carl Burch (1999), [http://mng.bz/c1i2](http://mng.bz/c1i2).

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After calling `hanoi()`, you should examine towers A, B, and C to verify that the discs were moved successfully.

```python
if __name__ == '__main__':
    hanoi(tower_a, tower_c, tower_b, num_discs)
    print(tower_a)
    print(tower_b)
    print(tower_c)
```

You will find that they were. In codifying the solution to the Towers of Hanoi, we did not necessarily need to understand every step required to move multiple discs from tower A to tower C. But we came to understand the general recursive algorithm for moving any number of discs, and we codified it, letting the computer do the rest. This is the power of formulating recursive solutions to problems—we often can think of solutions in an abstract manner without the drudgery of negotiating every individual action in our minds.

Incidentally, the `hanoi()` function will execute an exponential number of times as a function of the number of discs, which makes solving the problem for even 64 discs untenable. You can try it with various other numbers of discs by changing the `num_discs` variable. The exponentially increasing number of steps required as the number of discs increases, is where the legend of the Towers of Hanoi that you can read more about in any number of sources comes from. You may also be interested in reading more about the mathematics behind its recursive solution: See Carl Burch’s explanation in “About the Towers of Hanoi,” http://mng.bz/c1j2.

## 1.6 Real-world applications

The various techniques presented in this chapter (recursion, memoization, compression, and manipulation at the bit level) are so common in modern software development that it is impossible to imagine the world of computing without them. Although problems can be solved without them, it is often more logical or performant to solve problems with them.

Recursion, in particular, is at the heart of not just many algorithms, but even whole programming languages. In some functional programming languages, like Scheme and Haskell, recursion takes the place of loops in imperative languages. It is worth remembering, though, that anything accomplishable with a recursive technique is also accomplishable with an iterative technique.

Memoization has been applied successfully to speed up the work of parsers (programs that interpret languages). It is useful in all problems where the result of a recent calculation will likely be asked for again. Another application of memoization is in language runtimes. Some
language runtimes (versions of Prolog, for instance) will store the results of function calls automatically \((auto\text{-memoization})\), so that the function need not execute the next time the same call is made. This is similar to how the \texttt{@lru\_cache()} decorator in \texttt{fib6()} worked.

Compression has made an internet-connected world constrained by bandwidth more tolerable. The bit-string technique examined in section 1.2 is usable for real-world simple data types that have a limited number of possible values for which even a byte is overkill. The majority of compression algorithms, however, operate by finding patterns or structure within a data set that allow for repeated information to be eliminated. They are significantly more complicated than what is covered in section 1.2.

One-time pads are not practical for general encryption. They require both the encrypter and the decrypter to have possession of one of the same keys (the dummy data in our example) for the original data to be reconstructed, which is cumbersome and defeats the goal of most encryption schemes (keeping keys secret). But you may be interested to know that the name “one-time pad” comes from spies using real paper pads with dummy data on them to create encrypted communications during the Cold War.

These techniques are programmatic building blocks that other algorithms are built on top of. In future chapters you will see them applied liberally.

### 1.7 Exercises

1. Write yet another function that solves for element \(n\) of the Fibonacci sequence using a technique of your own design. Write unit tests that evaluate its correctness and performance relative to the other versions in this chapter.

2. We saw how the simple \texttt{int} type in Python can be used to represent a bit string. Write an ergonomic wrapper around \texttt{int} that can be used generically as a sequence of bits (make it iterable and implement \texttt{__getitem__()}). Reimplement \texttt{CompressedGene} using the wrapper.

3. Write a solver for The Towers of Hanoi that works for any number of towers.

4. Use a one-time pad to encrypt and decrypt images.